

RESEARCH ARTICLE

Manifolds and Catastrophes for Physical Systems

H. E. Gudrun Kalmbach

MINT, PF 1533, D-86818 Bad Woerishofen, Germany

Received on: 01-12-2020; Revised on: 19-02-2021; Acceptance on: 20-03-2021

ABSTRACT

New geometrical considerations are requested recently from physics. This arises after the finding of gravitational waves, demonstrating the length contraction/expansion as space-time ripples. In earlier publications, the author has published such geometries, dynamics, and shapes for physical systems, adding different kinds of mathematics. In this article, the focus is on manifolds which locally are like a real or complex Euclidean 4-dimensional (Hilbert) space H^4 . The influence of potential functions from catastrophe theory is investigated. They are a r -parameter family of smooth functions $R^n \rightarrow R$ for natural numbers n and all $r \leq 5$. There are seven catastrophes in Thom's theory which are structurally stable.^[10] Besides, the catastrophe manifolds other manifolds arise, the non-critical case without polar singularities and the non-degenerate critical Morse functions case with a diagonal matrix for the metric $\text{diag}(I_j - I_{n-i})$, I the identity matrix, $j \leq n$. Duals of catastrophes are not mentioned. Complex numbers, quaternion, and octonion vectorial extensions are added to H^4 . They are not used for the subspace lattice L of H^4 which remains 4-dimensional. L is the union of its Boolean blocks for sets of commuting projection operators $H^4 \rightarrow U_p$ which split $H^4 = U + U^\perp$, U^\perp the orthogonal complement of the closed subspace U . The block structure requests several catastrophes which are discussed in the first section. Applications are then added for bifurcations, Gleason measuring spin-like frames and for systems such as particles, quasi-particles, or effects on states of systems.

Key words: Manifolds, Catastrophes, Physics**FOLDS**

There is some research for unifying all energy systems as a Grand Theory. For the catastrophes, the Morse part (M) is not listed.^[1-5]

The folds $u_1^2 + t_1 u_1$, t_1 a parameter, u_j variables are for bifurcations. As basic energy which provides no unification theory, but has an evolution as in biology, is suggested a unified potential field POT with quarks replacing field quantum. They carry an electrical charge and mass with two potential functions $-b/r$, b a constant. As third energy, they have a color charge of quantum chromodynamics. It is assumed that they exist in a black hole 1-dimensional retracted to a central lemniscate while in H^4 , they are solid 3-dimensional (and radius extended in length) brezels of genus 2 (handle body with two

toroidal handles). From the POT treatment of other authors is accepted that it is a projective 5-dimensional extended field.^[6-10] A projector maps the field down to three 4-dimensional spaces which are projected into H^4 . The fields are with it in superposition as an electromagnetic and a gravitational potential field. The third scalar field can be Higgs to which physics attributes setting mass of systems at suitable barycenters. One or two dark matter or black hole or hitting galaxies decaying systems in motion arise(s) at the tip of a fold and bifurcate(s), disregarding Higgs as third option.

The further folds bifurcations are from EM(pot) to electrical fields charged leptons 1 as Cooper pair with magnetic fields quantum 4, including induction 5 between them as real 3-base vectorial cross product extension GF, noted in octonion coordinate indices as 145.^[11-14]

$E(\text{pot})$ (126 GF *rgb*-graviton) bifurcates in 2 time t functions, circular angular $E(\text{rot})$ and linear kinetic $E(\text{kin})$ energies. As speeds they are measured as $v = \Delta x / \Delta t$, x linear space coordinate and $\omega = d\phi /$

Address for correspondence:

H. E. Gudrun Kalmbach, MINT, PF 1533,
 D-86818 Bad Woerishofen, Germany.
 E-mail: mint-01@maxi-dsl.de

dt, ϕ polar spherical complex angle. The GF 356 is used for the dynamical SI rotor.^[12] Vectorials (eigenvectors of matrices) are angular momentum $L = r \times p$, r radius, and momentum $p = mv$, m mass, v linear speed. An orbital geometry 347 for $E(\text{rot})$ is a central normal oriented axis and a system rotating with p on a flat circle of radius r about the axis, for $E(\text{kin})$, it is a world line for a wave package, moving linear with p in space-time. There can be quasi-particles added to the (Eigen) vectors, for instance an orbiton or roton for $E(\text{rot})$ and a new introduced *kineton* for momentum which is not in the list for quasi-particles. Its optical computed group speed for a wave package requires a special relativistic mass m rescaling where frequency energy f of $E(\text{kin})$ is measured as mass $mc^2 = hf$, c speed of light, h Planck constant. A wavelength λ is also replaced by momentum in $\lambda p = h$ for the matter wave. Wave functions ψ use an electromagnetic interactions EMI octonion coordinate 7, 367 is a GF for p as octonion frequency coordinate 6 and 3 for a wave world line in direction of the space z -coordinate 3. The complex coordinate is in quaternions $z_1 = z + ict$ with time t as octonion 4 scaled and imaginary. For $E(\text{rot})$, the quaternion coordinates are $z_2 = x + iy$, in an 12 xy -plane and the real cross product z of x with y is for the rotation axis.

SI strong interaction with 8 gluons is generated, having the 8-dimensional $SU(3)$ symmetry. Its manifold is a twisted fiber bundle product $S^3 \times S^5$, locally a topological product of a 3- with a 5-dimensional unit sphere in real R^4 and complex C^3 . SI separates from gravity GR and POT. In addition, the *rgb-graviton* whirl is added to the gluons which are superpositions of 2 or 6 color charge whirls.

The heat chaos with GF 246 can have a characteristic polynomial of order 2 for fractals. The many fractal figures and a possible fractal theory exist. The dynamics is complicated, fractons are quasi-particles for this. The GF is for scaled temperature as product of a 3-dimensional spatial volume with pressure on the volumes surface. Acoustic phonons are the field quantum. As bubbles they contain some entropy inside the volume which cannot reach the absolute value 0 for heat, they decay before that. Most used *field* quantum such as *leptons*, *quarks*, and *gluons* as particles or bosons have some entropy attached.

If a series for phonons is wanted, the frequency musical scale can be used in form of intervals on base c as cd, ce, cf, cg, ca, cb or as scale c, d, e, f, g, a, b, h .

An exploding dark matter or black hole has generated space-time coordinate, measuring GFs for energies, systems listed as field quantum and fields (electrical EM as spin Eigen rotation 123, mass/potential GR 126 and Higgs bosons, orbiton 347, kineton 367, magneton 145, and heat 246) with quantized energy carriers, coming in series through a compass 07 and symmetries. The basic standard model has for a unified symmetry $U(1) \times SU(2) \times SU(3)$ for EMI's $U(1)$ circle, the quaternionic WI weak interaction $SU(2)$ and the $SU(3)$ for SI. As new symmetries are suggested complex Moebius transformations of a Riemannian sphere S_p for GR, dihedral symmetries D_n , $n = 0, 1, mm, n$, natural numbers with the n th roots symmetry as subgroup or integers Z as symmetry which are measured and quantized through complex winding numbers (a residual contour integration of $1/z$) for frequencies $f = 1/\Delta t = n$ with circulation time $\Delta t = 1/n$. A model in the MINT Wigris Tool bag uses tetrahedrons which have the S_4 permutation group of 4 elements while its factor group D_3 is used as nucleons quark triangle symmetry. The normal CPT Klein group of order 4 makes this and its factor classes have four elements: A symmetry, a coordinate, an energy, and a series for an energy. Series of 1, 2, 3, 4, and 6 are known for self-dual systems like a majorana, oscillon,

rgb-graviton, or soliton; cooper pairs exciton (broken Cooper pairs bogoliubon) for a series of 2, trion (nucleons) for a series of 3 (also for a kg GF of

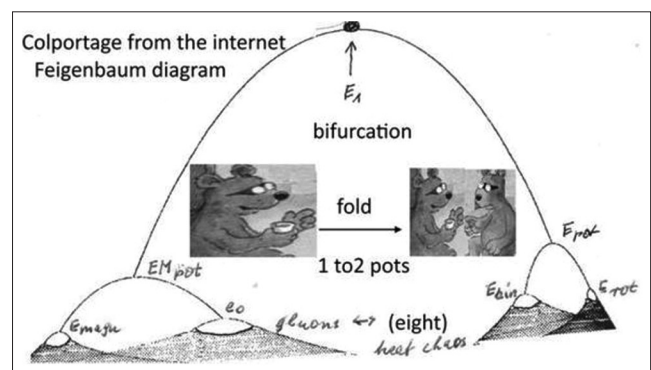


Figure 1: From POT the electrical and gravitational fields EM(pot), E(pot) bifurcate. EM(pot) has electrical charged leptons replacing field quantum and E(pot) *rgb-graviton* whirls as superposition of three conic color charge whirls red, green, and blue charges, using 6th roots of unity.

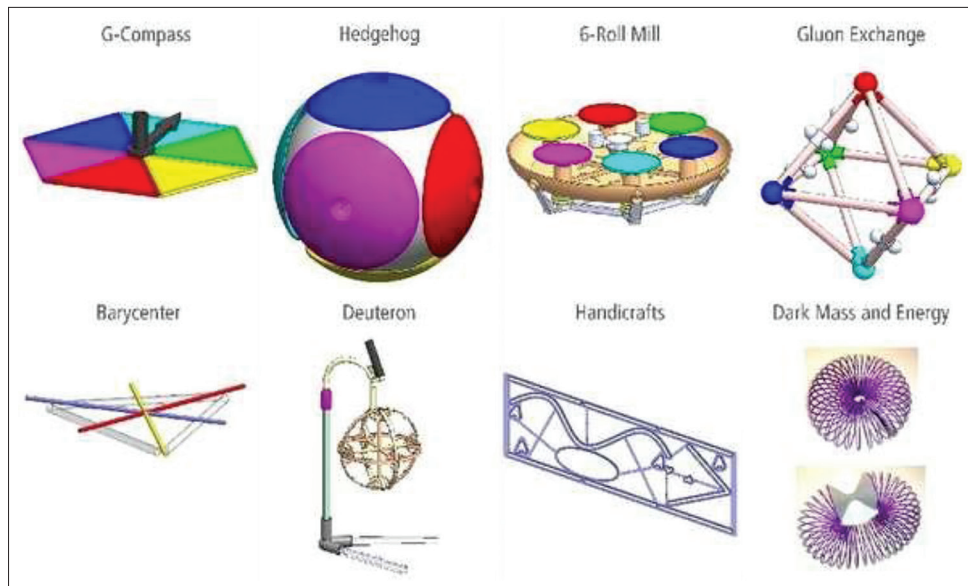


Figure 2: Tool bag with 8 models

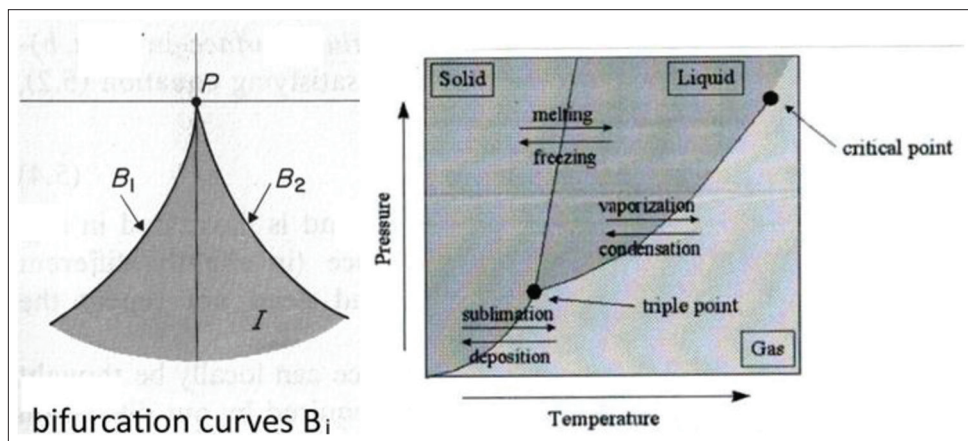


Figure 3: Cuspoid bifurcation, there is a cubic involved with three sheets upper, lower, and middle part, drawn at left as gray area between the bifurcation curves; triple points for the three sheets are shown at right for water as gas, liquid, and solid^[16,17]

leptons), EM charge; magnetic momentum, neural charge, momentum for a series of 4; many series of 6 (color charges, EM charges, six energies, a kg-GF for quarks complex weighted masses,d,) For the series, different kinds of a geometrical compass have been defined.^[9] In the MINT-Wigris Tool bag is a G-compass for color.

CUSPS AND THE ELLIPTIC UMBILIC

The Zeeman gravitational machine [Figure 4] demonstrates sudden changes, jumps of catastrophe maps and their derivatives. Its description and construction are found in Poston and Stewart.^[2-5] The cusp is the second catastrophe for it, +1 or -1 scaled $u_1^2 + t_2 u_2^2 + t_1 u_1$, t_j parameters, u_j variables. The elliptic umbilic is used for the 6 roll mill [Figure 2]. Its function is technically demonstrated

by 6 rolls driven by 3 motors [Figure 4]. For a quark-gluon plasma flow inside a nucleon, it has a water replacement. The three driving rotors for paired energy rolls are in color charge notation POT, driving red, turquoise, SI driving green, magenta and WI driving blue, yellow. The motors POT and SI have the same speed, the WI speed is different. The pairing in coordinates is according to the Heisenberg rays (for position-momentum, angle-angular momentum, and energy-time) on signed. xyz-space directions. In Figure 2, the part gluon exchange shows the linear pairing of rays. There is a weak rotor, described by Euler angles replacing the three coordinates which generate quaternions three Pauli matrices. In the wheel model are shown three wheels/rolls which are driven by water splashed on the shovels. In turning, they generate the coordinates as rotation axes.

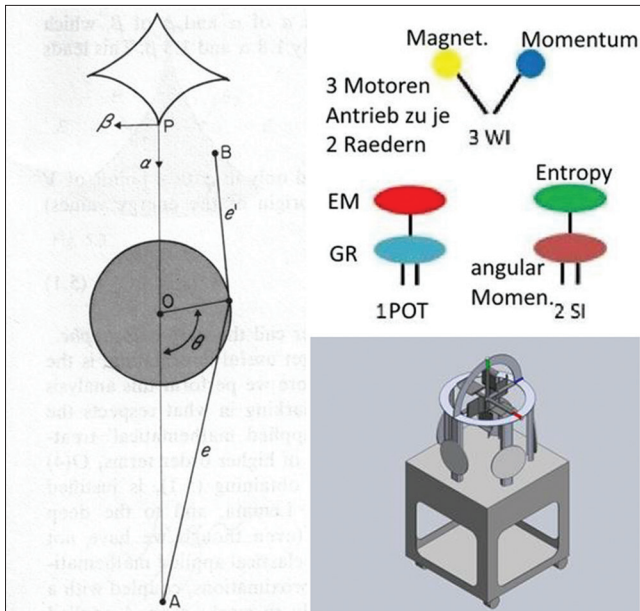


Figure 4: Zeeman machine which makes jumps when rotating the middle disk by a mechanical stretching, squeezing device (rubber bands as connections); 3 driving rotors for the 6 roll mill at upper right; wheel lower right

In a solid state, the 6 roll mill is demonstrated in the Tool bag as two fusion states [Figure 5]. For two protons, Higgs sets a common barycenter in the center of the two nucleon tetrahedrons with tip in the center and base the quark triangle of the nucleon, small color charge balls at the ends of the xyz-coordinates. Since on one coordinate a Cooper pair of uu quarks exists, a weak fusion decay is emitting from one u-quark an electron and an antineutrino, the tetrahedrons are rotating and the pairing on each coordinate is then between a u- and a d-quark (part at right in Figure 5). The inner dynamics for deuteron is a cyclic weak isospin exchange between the ud-Cooper pairs such that the proton and neutron are exchanging their location in space. The positron for a proton is drawn as two caps on the tetrahedron sides. If in atomic kernels the two tetrahedrons are in distance, but interacting through an exciton, nuclear decays occur when the distance becomes too large.

In a survey for the seven catastrophes, their equations show in a flow chart how they are connected from the most complicated parabolic umbilic or butterfly down to the simpler catastrophes [Figure 6]. As a research project is suggested to invent examples for the quanta range of the parabolic and hyperbolic umbilics, the butterfly, and the swallowtails.

The butterfly has been used in catastrophe theory for computing the stability of ships.^[10] For the

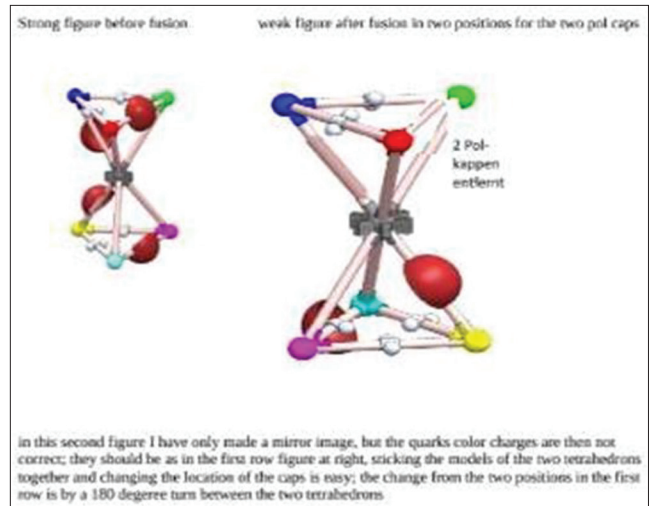


Figure 5: Fusion states for two protons at left, a deuteron with a neutron and a proton at right

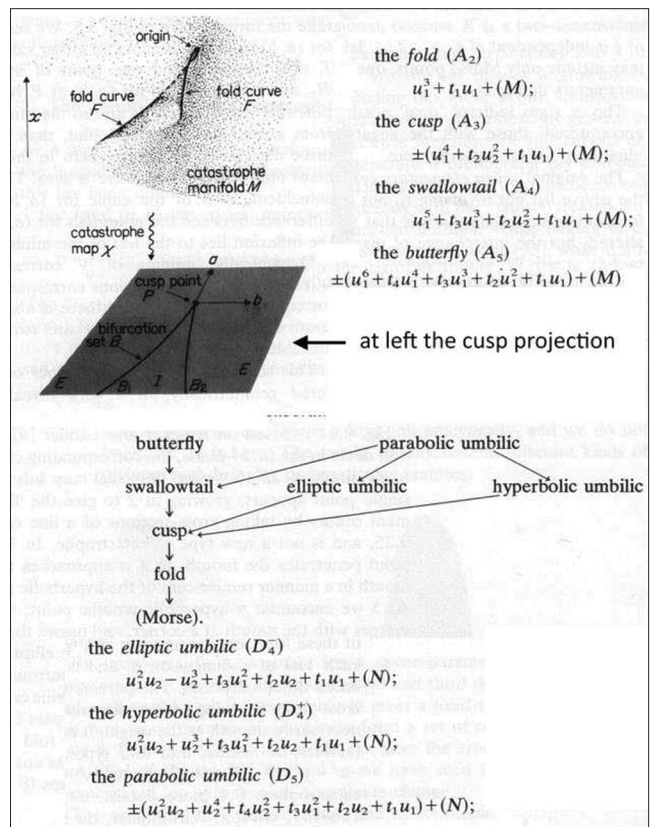


Figure 6: The cusp at left, the catastrophe equations at right, and in the middle in a flowchart how they connect

use of a hyperbolic umbilic, the article^[13] can be consulted. Parabolic and symbolic umbilics are studied in Thompson and Gaspar.^[14] Some interesting catastrophe applications are found in Thompson and Hunt, Bell and Lavis, Stanley, Wald, Howard, Chillingworth, Berry.^[15-21]

The research for a quasi-particles theory has not yet started. The GFs as metrical transformations can be added to them. Some field lines and quadrics for flows are from Poston and Stewart^[10] in Figure 7.

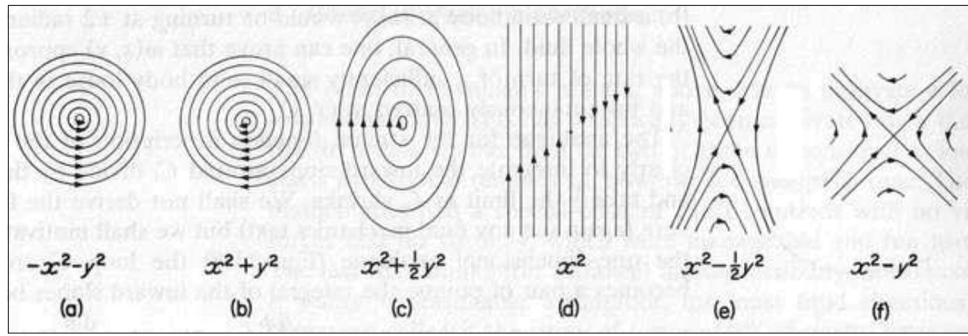


Figure 7: (Circular closed field lines) potential lines, (magnetic) vector field expansions, 2 roll mill, 4 roll mill flows; as liquids, they are rotating like a rigid body with speed proportional to the radius, as a wheel turning on its axis

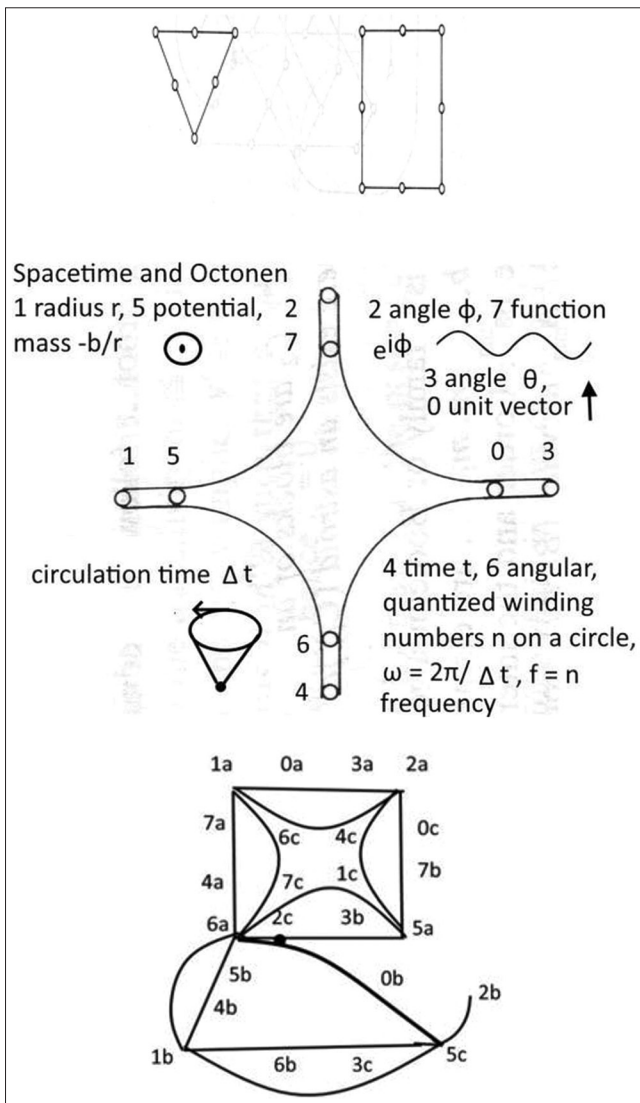


Figure 8: Flash with four Boolean blocks in L 1527, 1546, 0327, 0346; another L diagram (lower right) is with a 3-cycle; for this the requirement is that its vertices are in additional block, adding a fourth (mathematical) atom; the spin GF 123 for space coordinates requires time for a block xyzt (Eigen rotation of systems); the blocks are Boolean 24 overlapping for the flash in two atoms and intervals represent the blocks with 4 atoms marked on them; at upper right in 3 dimensions, Hi allows as lattices sub-diagram no 3- or 4-cycles; what that means for the GF is not clear, they can arise in superpositions having one atom in common

In a subspace lattice L diagram of H^4 for (f), the flash diagram of a coordinate bifurcation is quoted.^[1,6,8,11] When a 4-cycle in L exists, it requires the existence of four more Boolean blocks in a flash diagram. The doubling bifurcation is for octonion coordinates 0,ord from real space-time R^4 1234 to complex C^4 octonion coordinates in the combinations of Heisenberg uncertainties 15, 46 and newly bifurcating 03, 27 for a G-compass [Figure 2] and the extension of real to complex coordinates by the exponential polar function in $\phi \rightarrow \exp(i\theta)$ where linear complex/imaginary coordinates $x + iy$ are written in polar coordinates. The complex extension uses the second Pauli matrix as symmetry and the conjugation operator of physics [Figures 8 and 9].

As a final remark are listed some quasi-particles for a research project GF and quasi-particles: What is the Gleason operators measuring GF, geometry, dynamics, symmetry in meaningful models for anyon, (bi)polaron, dropletion (for fluids), exciton, fracton, holon (chargon), helicon, orbiton, oscillon, phason, plasmon, polaron, polariton, roton, soliton, trion, and wrinkleton (for cusps). For angles, a GF sLJ can include vectors for spin s , angular momentum L for orbits and a rotation axis $J = s + L$, as known from electrons orbits in an atoms shell, a polar(it)on? A soliton can have GR properties for ripples in space-time, changing metrically density as mass per volume $xyzm$ 1235; is this also for changing matter at the Schwarzschild radius of a black hole where their radius is (mathematically) inverted? Cusps are discussed earlier for decays with sudden changes of states and systems, wrinkletons fit to them as quasi-particles? Are dropletions for a fluid dynamics and triple points [Figure 3]? Oscillons are useful in the MINT-Wigris research project for

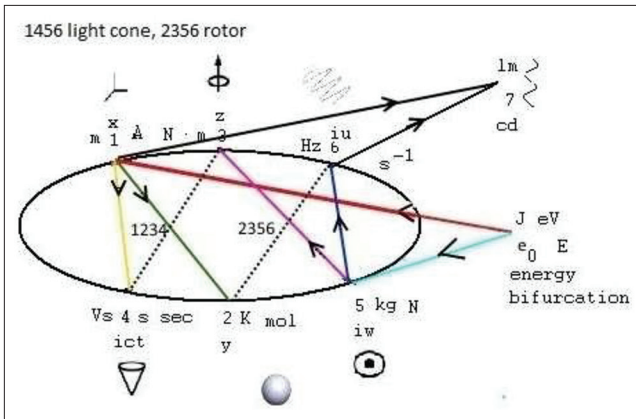


Figure 9: Pascal bifurcation for energies with a measuring GF triple and logos (1 spin triple, 2 heat volume, 3 rotational circle/orbit with axis, 4 magnetic whirl cone/time, 5 scalar mass at a barycenter of a system, 6 frequency as a cosine projection of an exponential function, and 7 as a helix line for EMI light expansions on a cylinders surface), e_0 as input is a units setting eigenvector for the measures and a compass needle of 07

the (SI) rotors, vectorial conic or wheel [Figure 4] rotations about axes. Plasmons can be used for the 6 roll mill. For the SI rotor, a side of the quark triangle is conic rotated through the gluon exchange between the sides quarks vertices P, Q where P is either red or green colored, rotates the side cw or mpo by 180 degrees and the oscillation surface (with oscillons?) is a half cone with three bounding circles after six rotations, perpendicular to the triangles plane. They form an oscillation with three quark knodes about the triangles circumference. Is this, a 1234 measuring device for functional integrations (radius dr or dx , time dt , area $dx dy$, volume $dx dy dz$)? The wheel [Figure 4] is a device for functional differentiation, the coupling in the Heisenberg uncertainties, $df(x)/dx$ 15, $dg(t)/dt$ 46 and (Euler) angular derivatives 23 are mostly used together with speeds as linear $v = \Delta x/\Delta t$ or angular $\omega = d\phi/dt$. Inverting a coordinate interval Δu to $1/\Delta u$ or d/du can use a quasi-particle, name one! For transformations serve the complex cross product invariant with its six D_3 values $z, 1/z, (1-z), 1/(1-z), (z-1)/z, z/(z-1)$ and $1/z$ makes inversions. For rotations serves $(z-1)/z$, for heat transfer $z/(z-1), -1/(z-1)$ is for $u = t$ time as frequency $f = 1/\Delta t$ and Δt or dt is for integrations with $(1-z)$. Find a quasi-particle presentation and theory for this!

Spinons and photons (helicons) are well studied and the MINT-Wigris research project has added to them the 123, 167 GF. Spin shows that the 3

dimensions xyz of a GF are in general to be extended to 4 dimensions xyzt and a Boolean block in the Hilbert space sub-lattice L. The rgb-graviton GF 126 is also extended to 1246, photons GF 167 is extended to 1467. Time developments are necessary, also for the dynamics. The 4-dimensional spaces can have a 5-dimensional projective extension (x_1, x_2, x_3, x_4, w) for 5-dimensional fields. In another Pascal bifurcation than in Figure 1, beside 1234 for space-time two more spaces 2356, 1456 arise as intersections of two Pascal intervals $14 + 23, 23 + 56$, and $14 + 56$. 14 is interpreted as a radius-time plane with the quadric $ru - cuadr$ 23 is interpreted as an angle/angular momentum quadric and 56 as an energy plane for mass m and frequency (m, f) with a line as degenerate case of a quadric $mca = hf$. This space is projective extended to (m, f, w) for the many gravitational, orbital quadrics. The GF metrical use of Gleason operators T for Hilbert space and other metrics is $\langle uT, u \rangle$. Projective normal forms (quadrics) are available, also for orbits, shapes, states, and catastrophes of systems. Quasi-particles contribute to this, how?

REFERENCES

1. Kalmbach G. Orthomodular Lattices. London, New York: Academic Press; 1983. p. 390.
2. Kalmbach HE. Mint-Wigris. Bad Woerishofen: MINT Verlag; 2017.
3. Kalmbach HE. (Chef-Hrsg), MINT (Mathematik, Informatik, Naturwissenschaften, Technik). Bad Woerishofen; MINT Verlag; 1997-2020. p. 1-65.
4. Internet Video under YouTube: Moebius Transformations Revealed; 2014.
5. Kalmbach G, Eberspaecher U. MINT-Wigris Tool Bag. Bad Woerishofen: MINT Verlag; 2019.
6. Kalmbach G. MINT-Wigris Postulates. In: Researchgate under MINT-Wigris Project; 2020.
7. Schmutzer E. Projektive einheitliche Feldtheorie. Frankfurt: Harry Deutsch; 2004.
8. Stierstadt K. Physik der Materie. Weinheim: VCH; 1989.
9. Kalmbach HE. MINT-Wigris Project in the Internet Researchgate; 2019.
10. Poston T, Stewart I. Catastrophe Theory and its Applications. London: Pitman; 1978.
11. Kalmbach HE, Heisenberg A. C-compass. J Appl Mater Sci Eng Res 2020;4:63-7.
12. Kalmbach HE. Nucleons Strong Interaction SI-Rotor, to Appear; 2017.
13. Thompson JM, Hunt GW. A bifurcation theory for the instabilities of optimization and design. In: Berlinski D, editor. Mathematical Methods in the Social Sciences. USA: researchgate; 1979.

14. Thompson JM, Gaspar ZA. A Buckling Model for the set of Umbilic Catastrophes. London: English Department University College; 1977.
15. Thompson JM, Hunt GW. General Theory of Elastic Stability. New York: Wiley; 1973.
16. Bell GM, Lavis DA. Thermodynamic Phase Changes and Catastrophe Theory, Preprint; 1978.
17. Stanley HE. Introduction to Phase Transitions and Critical Phenomina. London: Oxford University Press; 1971.
18. Wald A. On some systems of equations of mathematical economics. *Econometrica* 1957;19:368-408.
19. Howard IN. Bifurcations in reaction-diffusion problems. *Adv Math* 1975;16:246-58.
20. Chillingworth DR. The buckling beam. In: Manning A, editor. *Dynamical Systems*. Berlin: Springer; 1975. p. 86-91.
21. Berry MV. Waves and Thom's theorem. *Adv Phys* 1976;25:1-25.